

# Gauged M-flation, its UV sensitivity and Spectator Species

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## Abstract

In this paper we study gauged M-flation, an inflationary model in which inflation is driven by three  $N \times N$  scalar field matrices in the adjoint representation of  $U(N)$  gauge group. We focus our study on the gauged M-flation model which could be derived from the dynamics of a stack of D3-branes in appropriate background flux. The background inflationary dynamics is unaltered compared to the ungauged case of [1], while the spectrum of “spectator species”, the isocurvature modes, differs from the ungauged case. Presence of a large number of spectators, although irrelevant to the slow-roll inflationary dynamics, has been argued to lower the effective UV cutoff  $\Lambda$  of the theory from the Planck mass  $M_{\text{pl}}$ , putting into question the main advantage of M-flation in not having super-Planckian field values and unnaturally small couplings. Through a careful analysis of the spectrum of the spectators we argue that, contrary to what happens in N-flation models, M-flation is still UV safe with the modified (reduced) effective UV cutoff  $\Lambda$ , which we show to be of order  $(0.5 - 1) \times 10^{-1} M_{\text{pl}}$ . Moreover, we argue that the string scale in our gauged M-flation model is larger than  $\Lambda$  by a factor of 10 and hence one can also neglect stringy effects. We also comment on the stability of classical inflationary paths in the gauged M-flation.

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# 1 Introduction

The idea of inflation in one of its simplest realizations involves a massive scalar field with potential  $V = \frac{1}{2}m^2\phi^2$  whose mass has to be hierarchically smaller than Planck mass, *i.e.*  $m \simeq 6 \times 10^{-6} M_{\text{pl}}$ , where  $M_{\text{pl}} \equiv (8\pi G_N)^{-1/2} = 2.43 \times 10^{18}$  GeV is the reduced Planck mass. The amount of scalar field displacement in the field space needed to produced the required 60 e-folds of inflation in such model is much larger than Planck mass, explicitly  $\Delta\phi \simeq 14 M_{\text{pl}}$ . Moreover, recalling that the scalar field mass  $m$  is quadratically sensitive to the UV cutoff, one should explain the hierarchy between  $m$  and  $M_{\text{pl}}$ . The flatness of potential, *i.e.*  $m \ll M_{\text{pl}}$ , and super-Planckian field excursions  $\Delta\phi \gtrsim M_{\text{pl}}$  poses both theoretical and model building challenges to inflationary scenarios and models. For example, embedding of such a model in supergravity runs into various difficulties, since one has to guarantee the flatness of the potential on the scales which are beyond the limit of validity of the theory [2]. The super-Planckian field excursions are also troublesome recalling that  $M_{\text{pl}}$  is the ultimate UV cutoff in a theory coupled to (Einstein) gravity and hence one *may* worry about uncontrollable quantum corrections to the potential.<sup>1</sup> In supergravity or string theory motivated models one usually finds the size of the region in which inflation can happen to be around  $M_{\text{pl}}$  and it is therefore not possible to motivate the large field models like the one with quadratic potential, e.g. see [3]. It is, however, noteworthy that despite these model building problems, large field models are of phenomenological interest in anticipation of possible detection of B-mode polarization in the CMB, since they generally lead to sizeable primordial gravity waves.

Inspired by the idea of assisted inflation [5], one way out of the super-Planckian field excursion problem was examined in [6], where  $N$  scalar fields with polynomial chaotic-type potential cooperate to increase the Hubble friction and induce an inflation with enough number of e-folding. For example, for an assisted inflation model with  $N$  equal-mass scalar fields with quadratic potential,  $V = \frac{1}{2}m^2 \sum_{i=1}^N \phi_i^2$ , the N-flation [7], one finds that  $\Delta\phi_i \simeq 14M_{\text{pl}}/\sqrt{N}$ . Hence, by sufficiently increasing the number of scalar fields one can lower the amount of displacement of each field to below  $M_{\text{pl}}$ . Nonetheless, even in this approach, there is no justification for the hierarchy between  $m$  and  $M_{\text{pl}}$ . The situation is different for  $N$  identical decoupled scalar fields with polynomial potentials other than the quadratic one, in which the process of making the kinetic term canonical, scales the related couplings by negative powers of  $N$ . For cubic and quartic potentials, this will reduce the couplings by a factor of  $\sqrt{N}$  and  $N$ , respectively, *i.e.* the inflationary trajectory for an N-flation model with the potential  $V(\phi_i) = \sum_{i=1}^N \frac{\hat{\lambda}}{4}\hat{\phi}_i^4 - \frac{2\hat{\kappa}}{3}\hat{\phi}_i^3$  effectively behaves like a single field theory  $\phi$  with the potential  $V(\phi) = \frac{\lambda}{4}\phi^4 - \frac{2\kappa}{3}\phi^3$  where

$$\phi = \frac{\hat{\phi}}{\sqrt{N}}, \quad \kappa = \frac{\hat{\kappa}}{\sqrt{N}}, \quad \lambda = \frac{\hat{\lambda}}{N}. \quad (1.1)$$

One can thus justify the smallness of the effective couplings for the inflaton by increasing

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<sup>1</sup>Another point of view is that having a super-Planckian field values in itself does not mean that the quantum gravity effects are important. From this perspective, the super-Planckian energy density is responsible for triggering the quantum gravity effects and thus in a chaotic slow-roll inflation, with  $H \sim 10^{-5}M_{\text{pl}}$  and energy density way below  $M_{\text{pl}}^4$ , one may not worry about quantum gravity effect [4].

the number of scalar fields arbitrarily. For the case of quartic potential the value observed curvature perturbations, together with demanding 60 e-folds, gives  $\lambda \sim 10^{-14}$  and field excursions  $\Delta\phi \sim 10M_{\text{pl}}$  [8]. Therefore, requirement of having natural physical couplings, *i.e.*  $\hat{\lambda} \sim \mathcal{O}(1)$ , is met if  $N \sim 10^{14}$ . The value of physical field excursions is then  $\Delta\hat{\phi}_i \sim 10^{-6}M_{\text{pl}}$ .

Recently, motivated by the dynamics of  $N$  D3-branes subject to a proper RR six-form in a pp-wave background, an inflationary model was introduced [1, 9]. Due to the matrix nature of the inflatons, we dubbed this model as Matrix Inflation, or M-flation for brevity. In this model, three scalar fields corresponding to three dimensions perpendicular to the D3-branes play the role of the inflaton. In some specific representation for the matrices, the SU(2) sector, the dynamics of the system could be mapped to a single scalar field with a fourth order polynomial potential whose cubic and quartic couplings are lowered, respectively, by factors of  $N^{-3/2}$  and  $N^{-3}$  for large  $N$  [1]. The model is nonetheless not an assisted model, as in the assisted model all the scalar fields move in a concerted way to increase the Hubble friction and realize inflation. Thus, one should not expect the attractor behavior observed in assisted models [5, 10]. The model has this additional virtue of attaining the smallness of the couplings in generic chaotic inflation by a less number of degrees of freedom during inflation; this is achieved by  $N \sim 10^5$  for M-flation compared to  $N \sim 10^{14}$  of N-flation.<sup>2</sup>

The above advantages and successful features of N-flation, assisted inflation or M-flation is challenged by the claim of [11], stating that in the presence of  $N_s$  “light” species the universal gravitational cutoff is not  $M_{\text{pl}}$ ; it is  $\Lambda$ ,

$$\Lambda = \frac{M_{\text{pl}}}{\sqrt{N_s}}. \quad (1.2)$$

One may first make sure that the arguments of [11, 12] which has been mainly based on black hole physics considerations is also applicable to cosmological FRW setups. This we will argue for in section 3. In this case, nonetheless, as we will discuss only the modes with (effective) masses below the Hubble parameter contribute to the number of species  $N_s$ . For the case of  $m^2\phi_i^2$  N-flation model discussed above, where successful inflation implies  $m \sim 10^{-6}M_{\text{pl}}$ , with  $H \sim 10^{-5}M_{\text{pl}}$ , all of the  $N$  fields contribute to the species counting and hence  $N_s \simeq N$ . (A similar result is true for other chaotic N-flationary models discussed above.) This means that physical field excursions and the effective gravitational cutoff are lowered in the same way, by the factor  $1/\sqrt{N}$ . As such,  $\Delta\hat{\phi} \sim 10\Lambda$ . That is, although the naturalness problem for  $m/\Lambda$  –taking  $N \sim 10^{12}$ – or for the coupling  $\hat{\lambda}$  –taking  $N \sim 10^{14}$ – is solved, “larger-than-cutoff field excursion problem” is resurfaced again [10].

In this paper we revisit M-flation in view of the above lowered UV cutoff  $\Lambda$ . Some preliminary analysis in this direction has already been made in [10]. As we will show, unlike N-flation, M-flation is safe from the above mentioned UV problem. This is due to the specific feature of isocurvature modes of M-flation: the mass spectrum of isocurvature modes (which following [10] will be called “spectator modes”) of M-flation contains a variety of masses, with masses which are parametrically both above and below the Hubble parameter. Therefore,  $N_s$  does not scale like  $N^2$ . As we will show, for the value of parameters fixed by demanding

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<sup>2</sup>Note that in M-flation, where we deal with  $N \times N$  matrices, the number of degrees of freedom grows like  $N^2$ , which is still lower by four orders of magnitude than the N-flation number of degrees of freedom.

having a successful M-flation model,  $N_s$  is obtained to be of order 100-1000, while  $N \sim 10^5$ . This will save M-flation from the reappearance of the “larger-than-cutoff” problem.

The outline of this work is as follows. In section 2, we review the setup of M-flation. In this work, however, we consider a specific “gauged M-flation” model, which is more closely related to D-brane dynamics. We then compute the spectrum of spectators of the gauge M-flation model. In section 3, we review the arguments of [11] resulting in the UV cutoff modification (1.2) and extend those arguments to the cosmological FRW universe. In section 4, we confront gauged M-flation with the modified (lowered) gravitational UV cutoff and show that it is UV safe. The final section contains our concluding remarks and discussions. Some details of the arithmetics of the specific gauged M-flation model considered in this paper in a couple of inflationary regions has been gathered in the Appendix.

## 2 Gauged M-flation, the setup

M-flation, or Matrix inflation is the model in which inflation is driven by three  $N \times N$  hermitian matrices  $\Phi_i$  ( $i = 1, 2, 3$ ) as inflaton fields with a specific quartic potential [1, 9]. The M-flation model of [1] has a *global*  $U(N)$  symmetry and  $\Phi_i$  are in its adjoint representation. On the other hand,  $N$  string theory D3-branes probing specific background geometry provides a natural setting in which M-flation can be realized. In the brane theory setting, however, this  $U(N)$  is gauged. In this work, motivated by the string theory picture, we will focus on the “gauged M-flation model” the action for which is

$$S = \int d^4x \sqrt{-g} \left( \frac{-M_P^2}{2} R - \frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{1}{2} \sum_i \text{Tr}(D_\mu \Phi_i D^\mu \Phi_i) - V(\Phi_i, [\Phi_i, \Phi_j]) \right), \quad (2.1)$$

where the signature of the metric is  $(-, +, +, +)$ ,  $D_\mu$  is the gauge covariant derivative and  $F_{\mu\nu}$  is the gauge field strength:

$$D_\mu \Phi_i = \partial_\mu \Phi_i + ig_{YM} [A_\mu, \Phi_i], \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig_{YM} [A_\mu, A_\nu], \quad (2.2)$$

and the Tr is over  $N \times N$  matrices. The potential  $V(\Phi_i, [\Phi_i, \Phi_j])$  can be motivated from dynamics of  $N$  D3-branes subject to an RR six-form, whose strength is parameterized by  $\hat{\kappa}$  and has two legs along the directions transverse to the D3-branes, in a specific ten-dimensional IIB supergravity background [13]:<sup>3</sup>

$$ds^2 = -2dx^+ dx^- - \hat{m}^2 \sum_{i=1}^3 (x^i)^2 (dx^+)^2 + \sum_{I=1}^8 dx_I dx_I, \quad (2.3)$$

$$C_{+123ij} = \frac{2\hat{\kappa}}{3} \epsilon_{ijk} x^k. \quad (2.4)$$

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<sup>3</sup>Strictly speaking, in order to view (2.1), with a nonvanishing four dimensional Planck mass, as the low energy effective theory of  $N$  D3-branes in the background (2.3), we need to demand the six dimensional transverse space to be compact. This could be achieved if we considered a background geometry which around  $x^i \sim l_s$  behaves like (2.3) and at large values of  $x^i$  becomes a (Ricci) flat geometry, which is then compactified on a  $T^6$  or  $CY_3$ . This latter is similar to the standard KKLMNT scenario [14] where the  $AdS_5 \times S^5$  throat is completed into an  $R^4 \times CY_3$ .

The matrices  $\Phi_i$  are proportional to three out of six dimensions transverse to the D3-branes

$$\Phi_i \equiv \frac{X_i}{\sqrt{(2\pi)^3 g_s l_s^2}}, \quad (2.5)$$

and the potential takes the form

$$V = \text{Tr} \left( -\frac{\lambda}{4} [\Phi_i, \Phi_j] [\Phi_i, \Phi_j] + \frac{i\kappa}{3} \epsilon_{jkl} [\Phi_k, \Phi_l] \Phi_j + \frac{m^2}{2} \Phi_i^2 \right), \quad (2.6)$$

where  $i = 1, 2, 3$  and we are hence dealing with  $3N^2$  real scalar fields. Here and below the summation over repeated  $i, j$  indices is assumed.  $\lambda$  and  $\kappa$  are related to the string coupling and the strength of the Ramond-Ramond antisymmetric form and  $m$  is the same  $\hat{m}$  that appears in the metric:

$$\lambda = 8\pi g_s = 2g_{YM}^2, \quad \kappa = \hat{\kappa} g_s \sqrt{8\pi g_s}, \quad m^2 = \hat{m}^2. \quad (2.7)$$

From the string theory perspective, we need to choose  $\hat{m}^2$  and  $\hat{\kappa}$  such that (2.3) is a solution to supergravity equation of motion with a constant dilaton, *i.e.*

$$\lambda m^2 = 4\kappa^2/9. \quad (2.8)$$

In [1], we relaxed the above relation between the parameters of the potential and construct more general M-fflation models by treating  $\lambda$ ,  $\kappa$  and  $m^2$  as independent parameters. In this work we will restrict ourselves to gauged M-fflation models with (2.8) relation between its parameters.

## 2.1 Background inflationary trajectory

The equations of motion for the scalar and vector fields is given by

$$\begin{aligned} D_\mu D^\mu \Phi_i + \lambda [\Phi_j, [\Phi_i, \Phi_j]] - i\kappa \epsilon_{ijk} [\Phi_j, \Phi_k] - m^2 \Phi_i &= 0, \\ D_\mu F^{\mu\nu} - i g_{YM} [\Phi_i, D^\nu \Phi_i] &= 0. \end{aligned} \quad (2.9)$$

As discussed in [1], for the ungauged case, one can consistently restrict the classical dynamics to a sector in which we are effectively dealing with a single scalar field  $\hat{\phi}$ . This sector, which will be called the  $SU(2)$  sector, is obtained for matrix configurations of the form

$$\Phi_i = \hat{\phi}(t) J_i, \quad i = 1, 2, 3, \quad (2.10)$$

where  $J_i$  are the basis for the  $N$  dimensional irreducible representation of the  $SU(2)$  algebra

$$[J_i, J_j] = i \epsilon_{ijk} J_k, \quad \text{Tr}(J_i J_j) = \frac{N}{12} (N^2 - 1) \delta_{ij}. \quad (2.11)$$

Since both  $\Phi_i$  and  $J_i$  are hermitian, we conclude that  $\hat{\phi}$  is a real scalar field.

It is straightforward to show that the  $[\Phi_i, D_\nu \Phi_i]$  term in the equation of motion of the gauge field for the ansatz (2.10) is proportional to  $[J_i, [J_i, A_\nu]]$  and hence vanishes for  $A_\mu = 0$ .

Therefore, in the  $SU(2)$  sector one can consistently turn off the gauge fields  $A_\mu$  in the background, *i.e.* the classical inflationary trajectory takes place in the scalar fields  $\Phi_i$  sector.<sup>4</sup>

Plugging these into the action (2.1) and adding the four-dimensional Einstein gravity, we obtain

$$S = \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R + \text{Tr} J^2 \left( -\frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{\lambda}{2} \hat{\phi}^4 + \frac{2\kappa}{3} \hat{\phi}^3 - \frac{m^2}{2} \hat{\phi}^2 \right) \right], \quad (2.12)$$

where  $\text{Tr} J^2 = \sum_{i=1}^3 \text{Tr}(J_i^2) = N(N^2 - 1)/4$ . Interestingly enough, this represents the action of chaotic inflationary models with a non-standard kinetic energy. Upon the field redefinition

$$\hat{\phi} = (\text{Tr} J^2)^{-1/2} \phi = \left[ \frac{N}{4} (N^2 - 1) \right]^{-1/2} \phi, \quad (2.13)$$

the kinetic energy for the new field  $\phi$  takes the canonical form, while the potential becomes

$$V_0(\phi) = \frac{\lambda_{eff}}{4} \phi^4 - \frac{2\kappa_{eff}}{3} \phi^3 + \frac{m^2}{2} \phi^2, \quad (2.14)$$

where

$$\lambda_{eff} = \frac{2\lambda}{\text{Tr} J^2} = \frac{8\lambda}{N(N^2 - 1)}, \quad \kappa_{eff} = \frac{\kappa}{\sqrt{\text{Tr} J^2}} = \frac{2\kappa}{\sqrt{N(N^2 - 1)}}. \quad (2.15)$$

Depending on the choice of parameters,  $\lambda$ ,  $\kappa$  and  $m^2$ , several inflationary scenarios could be realized in Matrix inflation setup which was studied in [1]. In this work we will focus on the “symmetry breaking” case where these parameters are related as in (2.8). Our aim here is to study the effective excursions of the field with the species-reduced UV cutoff [11] and examine the UV stability of the M-flation scenario.

## 2.2 Spectrum of gauged M-flation “spectator” modes

In the gauged M-flation we start with three  $N^2$  scalar fields  $\Phi_i$  and  $4N^2$  fields  $A_\mu$ . Reducing to the  $SU(2)$  sector (2.10), we have turned on only one combination these fields at the background level. Out of the remaining  $7N^2 - 1$  fields, recalling the gauge symmetry of the action, we expect  $2N^2$  of the gauge fields and  $3N^2 - 1$  of the scalars to be physical; the  $2N^2$  of the gauge field degrees of freedom is removed by the equations of motion and gauge invariance. As we will show momentarily, although the total count of  $5N^2 - 1$  isocurvature modes remains intact, due to the (spontaneous) symmetry breaking induced by the potential  $V(\Phi_i)$ , we will have  $3N^2 - 1$  vector field degrees of freedom and  $2N^2$  scalar modes. These  $5N^2 - 1$  modes, however, can be excited quantum mechanically. As was discussed in [1] backreaction of these isocurvature modes on the inflationary background, at least during slow-roll period, is very small. Moreover, these isocurvature modes do not couple to the quantum fluctuations of the effective inflaton field  $\phi$ . We will hence, following [10], call these modes as *spectators*. The arguments of [1] are made for the ungauged case and, as we will show below, it is straightforward to extend those to the gauged case. We start our analysis by working out the *spectators* mass spectrum.

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<sup>4</sup>As discussed in [15] it is possible to construct inflationary models where certain combination of the non-Abelian gauge fields play the role of effective inflaton. For the latter, however, one should have a specific gauge field action.

**Spectrum of scalar spectators.** The analysis of the spectrum for these modes is the same as the ungauged case of [1], except for the fact that “zero-modes” are not physical in the gauged case (see below). To compute the spectrum of scalar fluctuations  $\Psi_i$ , defined as

$$\Phi_i = \hat{\phi} J_i + \Psi_i, \quad (2.16)$$

we expand the action to second order in  $\Psi$  while turning off the gauge fields, yielding

$$\mathcal{L}_\Psi^{(2)} = -\frac{1}{2}\text{Tr}(\partial_\mu \Psi_i)^2 + \text{Tr} \left[ \frac{\lambda}{2} \hat{\phi}^2 (\epsilon_{ijk} [J_j, \Psi_k])^2 + i \left( \frac{\lambda}{2} \hat{\phi}^2 - \kappa \hat{\phi} \right) \epsilon_{ijk} [J_i, \Psi_j] \Psi_k - \frac{m^2}{2} \Psi_i^2 \right]. \quad (2.17)$$

The above is “diagonalized” for  $\Psi_i$  satisfying

$$i\epsilon_{ijk} [J_j, \Psi_k] = \omega \Psi_i. \quad (2.18)$$

The solutions of the above are spin one (vector) representations of  $\text{SU}(2)$ . The details may be found in [16], here we only quote the result:

- *zero modes:*  $\omega = -1$

$$i\epsilon_{ijk} [J_j, \Psi_k] = -\Psi_i. \quad (2.19)$$

There are  $N^2 - 1$  of these modes. In the ungauged theory [1] these modes are physical, while in the gauged model they are not, as they are gauge degrees of freedom. To see the latter let us recall that under a global infinitesimal gauge transformation  $\Phi_i \rightarrow \Phi_i + ig[\Lambda, \Phi_i]$  where  $\Lambda$  is a generic  $N \times N$  hermitian matrix. Therefore, a  $\Psi_i$  in the zero mode is nothing but a gauge transformation over the background solution  $\Phi_i = \hat{\phi} J_i$ . We hence discard these modes as unphysical in our current analysis.

- $\alpha_j$ -modes:  $\omega = -(j+2)$  and  $0 \leq j \leq N-2$ . Degeneracy of each  $\alpha_j$  mode is  $2j+1$ . There is therefore,  $(N-1)^2$  of these modes. Recalling (2.17), the mass of these modes are

$$M_{\alpha_j}^2 = \frac{1}{2} \lambda_{eff} \phi^2 (j+2)(j+3) - 2\kappa_{eff} \phi (j+2) + m^2. \quad (2.20)$$

- $\beta_j$ -modes:  $\omega = j-1$  and  $1 \leq j \leq N$ . Degeneracy of each  $\beta_j$ -mode is  $2j+1$  and hence there are  $(N+1)^2 - 1$  of  $\beta$ -modes. Mass of  $\beta_j$  mode is

$$M_{\beta_j}^2 = \frac{1}{2} \lambda_{eff} \phi^2 (j-1)(j-2) + 2\kappa_{eff} \phi (j-1) + m^2. \quad (2.21)$$

**Spectrum of gauge field spectators.** To read the spectrum of the gauge fields we turn on  $A_\mu$  and expand the action (2.1) to second order in  $A_\mu$  while replacing for the value of  $\Phi_i = \hat{\phi} J_i$ . The second order gauge field action is then obtained to be

$$\mathcal{L}_{A_\mu}^{(2)} = -\frac{1}{4} \text{Tr}(\partial_{[\mu} A_{\nu]})^2 + \frac{1}{2} g_{YM}^2 \hat{\phi}^2 \text{Tr}([J_i, A_\mu][J_i, A_\mu]). \quad (2.22)$$

The mass spectrum can be read from the second term recalling that  $\text{Tr}([J_i, A_\mu][J_i, A_\mu]) = \text{Tr}(A_\mu[J_i, [J_i, A_\mu]])$  and that  $[J_i, [J_i, X]] = \omega X$  eigenvalue problem has eigenvalues  $j(j+1)$

with  $j = 0, \dots, N - 1$  and degeneracy of each mode is  $2j + 1$ . Therefore, we are dealing with a system of massive vector fields with masses

$$M_{A,j}^2 = \frac{\lambda_{eff}}{4} \phi^2 j(j+1). \quad (2.23)$$

As we see  $j = 0$  mode is massless and corresponds to the  $U(1)$  sector in the  $U(N)$  matrices. Degeneracy of the vector field modes is hence  $3(2j + 1)$  for  $j \geq 1$  modes and is two for  $j = 0$  mode. (The factor of three has appeared due to the three polarizations of each massive vector field.) We hence have  $3N^2 - 1$  vector field modes.

In summary we have  $(N - 1)^2$   $\alpha$ -modes,  $N^2 + 2N$   $\beta$ -modes and  $3N^2 - 1$  vector field modes, altogether  $5N^2$  modes. The  $\alpha_{j=0}$  mode (which has degeneracy one) and mass  $M^2 = 6\lambda\hat{\phi}^2 - 4\kappa\hat{\phi} + m^2$  is the quantum fluctuations of the  $SU(2)$  sector scalar effective inflaton field  $\phi$  and is hence the adiabatic mode while all the other  $5N^2 - 1$  modes are “isocurvature”.

### 3 Effective gravity UV cutoff and number of species

Planck scale  $M_{\text{pl}}$  in the Einstein gravity has two roles: 1) Recalling that  $8\pi G_N = M_{\text{pl}}^{-2}$ ,  $M_{\text{pl}}$  is the coupling of *classical* gravity and, 2) the energy scale above which quantum gravity effects kick in; it is the ultimate UV cutoff for the quantum field theories above which (quantum) gravity effects cannot be ignored. There are, however, various perturbative and non-perturbative arguments suggesting that the UV cutoff  $\Lambda$  above which (quantum) gravity effects become important is not  $M_{\text{pl}}$ , and it can be a (much) lower scale [11, 12, 17]. Here we review some of these arguments. The perturbative argument is as follows: suppose we have  $N_s$  quantum fields with masses  $\Lambda$  coupled to gravity. Each of these quantum fields will induce a factor proportional to  $\Lambda^2$  into the renormalizable Planck mass [18, 19]. Modulo possible accidental cancelations, this suggests that the effective contribution to the Planck mass is proportional to  $N_s \Lambda^2$ . Of course this perturbative argument only suggests that  $N_s \Lambda^2 \leq M_{\text{pl}}^2$ . As it is clear,  $N_s$  is the number of species whose mass is below the cutoff scale  $\Lambda$ . This result receives backing from other known physics, which we will review below. In the non-trivial gravity backgrounds where there is an energy scale associated with the background itself, the question of which degrees of freedom contribute to the counting  $N_s$  should be revisited. Below we discuss two such cases: A (Schwarzschild) black hole and the FRW cosmological background.

**Black holes and the species cutoff.** It is an established fact that a Schwarzschild black hole of mass  $M_{BH}$  is a thermodynamical system with the Bekenstein-Hawking entropy [20, 21]

$$S_{BH} = 2\pi M_{\text{pl}}^2 A_h = \frac{1}{2} \left( \frac{M_{BH}}{M_{\text{pl}}} \right)^2, \quad (3.1)$$

where  $A_h$  is the horizon area, and at Hawking temperature

$$T_{BH} = \frac{M_{\text{pl}}^2}{M_{BH}}. \quad (3.2)$$



As in any thermodynamical system, unitarity is lost *unless* there is an underlying unitary statistical mechanical description which leads to the thermodynamical system in the “thermodynamic limit”. Despite the partial progress for some special cases, a general statistical mechanical description of Bekenstein-Hawking entropy is still lacking. One such attempt is to understand the black hole entropy as the entanglement entropy of a system accounting for the “microstates” of the black hole. If this system consists of  $N_s$  number of species lighter than the cutoff scale  $\Lambda$  this entanglement entropy is [22]

$$S_{\text{ent}} = N_s \frac{\Lambda^2}{M_{\text{pl}}^2} A_h M_{\text{pl}}^2. \quad (3.3)$$

Assuming  $\Lambda \simeq M_{\text{pl}}$  leads to species problem: even though the Bekenstein-Hawking entropy is universal, the entanglement entropy is not and depends on the number of species. Eq.(1.2) can serve as a resolution to this problem.

The above argument is not limited to interpreting the Bekenstein-Hawking entropy as the entanglement entropy and can be argued for noting the thermodynamical nature of the black hole. To see this more clearly let us recall that the Hawking radiation of a black hole consists of particles which it can thermally produce, *i.e.* their masses are less than its Hawking temperature (3.2). On the other hand the semi-classical treatment of black hole is only valid for  $T_{BH} \lesssim \Lambda$  and when its energy emission rate  $dE/dt \lesssim M_{\text{pl}}^2$ . The energy emission rate is proportional to  $N_s T_{BH}^4 A_h$ , where  $N_s$  is the number of (relativistic) particles which can be produced by the black hole. Putting these together, we learn that  $N_s \Lambda^2 \lesssim M_{\text{pl}}^2$ .

One may also argue for this bound in a different way: Let us suppose that we have a system of  $N_s$  quantum fields of mass  $m_0$  (we are assuming that  $m_0$  is less than the eventual cutoff  $\Lambda$ ) and that these  $N_s$  fields are labeled by e.g. a discrete  $Z_{N_s}$  symmetry. Semiclassical description for a black hole is available if its Hawking temperature is at most of order the smallest mass state available, *i.e.*  $T_{BH} \gtrsim m_0$ . Moreover, due to no-hair theorem, the black hole state should be  $Z_{N_s}$  invariant and hence its lowest mass is  $N_s m_0$ . These again imply that  $m_0^2 \lesssim M_{\text{pl}}^2/N_s$ . It is worth noting that this arguments are compatible with the physical expectation that the life-time of a semiclassical black hole should not be less than  $\Lambda^{-1}$ .

**FRW backgrounds and the species cutoff.** For the case of a black hole horizon, our argument was mainly based on the fact that there is a natural energy scale associated with the system, the Hawking temperature  $T_{BH}$  and that the effective cutoff  $\Lambda$  must be less than this temperature. For the cases where we have a cosmological horizon this natural scale should be replaced with the Hubble parameter of the space  $H$ . In a classic paper Gibbons and Hawking [23] have argued that for the cosmological event horizons indeed one can still use the “first law of black hole thermodynamics” in the same way as used for black hole event horizons, but with  $T_{BH}$  replaced  $\kappa_G/(2\pi)$  where  $\kappa_G$  is the surface gravity at the cosmological event horizon, which is nothing but the Hubble parameter  $H$ .

The above has a direct manifestation in the well-established cosmic perturbation theory, e.g. see [24]: the amplitude of quantum fluctuations of “light” fields, fields whose mass are small compared to the Hubble radius  $H$ , at the horizon crossing, is equal to the value set by the “thermal” fluctuations, which is nothing but the Gibbons-Hawking temperature. In

other words in a cosmological setting species which contribute to the cutoff are number of the fields whose mass is less than the Hubble parameter. This is what we are going to use next to compute the effective cutoff for the M-flation.

## 4 Gauged M-flation and the species gravitational UV cutoff

Having worked out the spectrum of the gauged M-flation spectators we are now ready to calculate the number of species  $N_s$  contributing to the effective cutoff  $\Lambda$ :

$$N_s = \text{number of spectators with mass less than } H, \quad (4.1)$$

where  $H$  is given by the Friedmann equation

$$3H^2 M_{\text{pl}}^2 = \frac{\lambda_{eff}}{4} \mu^4 x^2 (x-1)^2. \quad (4.2)$$

In the above we have considered the “symmetry breaking” M-flation model (see the Appendix for a more detailed inflationary analysis of this case),

$$\phi = \mu x, \quad \mu^2 = \frac{2m^2}{\lambda_{eff}}.$$

Having a successful inflationary model implies that  $x$  is a parameter of order one during inflation,  $\mu$  is of order  $25 - 35 M_{\text{pl}}$  and  $\lambda_{eff} \sim 5 \times 10^{-14}$  [1], see also the Appendix here. In terms of  $x$  and  $\mu$  parameters the masses are

$$\begin{aligned} M_{\alpha_j}^2 &= \frac{\lambda_{eff} \mu^2}{2} [x^2(j+2)(j+3) - 3x(j+2) + 1], \quad 0 \leq j \leq N-2 \\ M_{\beta_j}^2 &= \frac{\lambda_{eff} \mu^2}{2} [x^2(j-1)(j-2) + 3x(j-2) + 1], \quad 1 \leq j \leq N \\ M_{A,j}^2 &= \frac{\lambda_{eff} \mu^2}{4} x^2 j(j+1), \quad 0 \leq j \leq N-1, \end{aligned} \quad (4.3)$$

The above mass spectra are increasing as we increase  $j$ ; for large  $j$  they grow like  $j^2$ . Noting that  $H^2 \gg \lambda_{eff} \mu^2$ , the heaviest mode which contribute to the species count is then given by

$$j_{max}^2 \simeq \frac{(x-1)^2}{6} \left( \frac{\mu}{M_{\text{pl}}} \right)^2 \quad (4.4)$$

for  $\alpha$  and  $\beta$  modes, and twice as much for the gauge field modes. Therefore, considering the degeneracy of state for a given  $j$ ,  $N_s$  is given by

$$N_s \simeq (2 + 3 \cdot 2) j_{max}^2 = \frac{4(x-1)^2}{3} \left( \frac{\mu}{M_{\text{pl}}} \right)^2. \quad (4.5)$$

In the above the factor of 2 is for  $\alpha$  and  $\beta$  modes and  $3 \cdot 2$  is for the gauge fields; 3 for the polarization of massive gauge fields and 2 for the extra factor of 2 in the spectrum of gauge fields compared to  $\alpha$ ,  $\beta$  modes (cf (4.3)). The effective UV cutoff is then

$$\frac{\Lambda^2}{M_{\text{pl}}^2} = \frac{1}{N_s} = \frac{3}{4(x-1)^2} \left( \frac{M_{\text{pl}}}{\mu} \right)^2, \quad \text{or} \quad \Lambda = \frac{2(x-1)}{\sqrt{3}} \frac{\mu}{N_s}. \quad (4.6)$$

We note that  $N_s$  and hence  $\Lambda$  only depend on  $\mu$  and not the other parameter of the model  $\lambda_{eff}$ , whereas size of the matrices  $N$  was fixed on the requirement of having an order one  $\lambda$  parameter. For a successful “natural” inflation (see the Appendix)  $N \sim 5 \times 10^4$  while  $N_s \sim (2 - 8) \times 10^2$ . As we see  $N_s$  is not only different than  $N^2$  as one would have naively thought, but also  $N_s \ll N$ . For our model, as we see, it happens that numerically  $N_s \sim \sqrt{N}$  and  $\Lambda \simeq 0.05 M_{\text{pl}}$ .

It is instructive to compare the scale of energy density during inflation with the cutoff scale, *i.e.*  $\rho/\Lambda^4$  ratio, where  $\rho$  is the energy density driving inflation. One can show that

$$\frac{\rho}{\Lambda^4} = \lambda_{eff} N_s^4 \frac{9x^2}{64(x-1)^2}. \quad (4.7)$$

For the parameters of our model this ratio is  $< 10^{-3}$ . We would like to stress that, although it happens for our model, the energy density of the background  $\phi$  field during inflation need not be less than  $\Lambda^4$ . This is due to the fact that, as can be seen from our discussions of the previous section, the suppression of the effective cutoff with respect to  $M_{\text{pl}}$  is only relevant to the quantum fluctuations and not the background classical fields.

As discussed the M-flation is motivated by or derived from dynamics of D-branes in string theory. The D-branes are, however, described by Born-Infeld and our M-flation action is obtained from expansion of the Born-Infeld in the the leading order in string scale  $m_s$ . In using M-flation, one should then make sure that i) keeping the first order terms in the Born-Infeld is a valid expansion and ii) the stringy effects should not become important below the cutoff scale  $\Lambda$ , *i.e.*  $m_s \gtrsim \Lambda$ . One would also physically expect  $m_s \leq M_{\text{pl}}$ . The ratio of the first two terms in the expansion of Born-Infeld action for  $N$  branes is given by

$$\delta \equiv \frac{\text{Tr}([\Phi_i, \Phi_j]^4)}{m_s^2 \text{Tr}([\Phi_i, \Phi_j]^2)} \sim \left(\frac{\hat{\phi}}{m_s}\right)^2 \frac{\text{Tr} J^4}{\text{Tr} J^2} \sim \frac{\phi^2}{4N m_s^2}. \quad (4.8)$$

In computing the above (2.10), (2.11) and (2.13) and,  $\text{Tr} J^n \sim N(N/2)^n$  approximation have been used. Moreover, in the above estimate for  $\delta$  in “ $\sim$ ” we are missing factors of order  $10^{-1}$ . A good estimate for  $\delta$  is obtained by replacing  $\phi$  by the value of the field at its minimum  $\mu$ . This gives a lower bound approximation for the  $\phi > \mu$  inflationary model and an upper bound for  $\phi < \mu$  hilltop inflation case. Demanding  $\delta \lesssim 1$  yields

$$m_s \gtrsim \frac{\mu}{\sqrt{N}} \quad \text{or} \quad m_s \gtrsim \Lambda \frac{N_s}{\sqrt{N}}. \quad (4.9)$$

For our case  $\frac{N_s}{\sqrt{N}} \sim 1$  and hence  $m_s \gtrsim \Lambda$ . Therefore, one can safely ignore the stringy corrections and the Born-Infeld corrections to the M-flation action once the energy remains below the cutoff  $\Lambda$ . With the numeric values for  $N$  and  $N_s$ ,  $\Lambda = 5 \times 10^{-2} M_{\text{pl}}$ ,  $m_s \sim 10^{-1} M_{\text{pl}}$  is a reasonable range.

## 5 Concluding remarks

We demonstrated that in the gauged M-flation, and in its  $\text{SU}(2)$  sector, the physical field excursion remains below the species UV cutoff  $\Lambda$ . This is rooted in the fact that species

which contribute to the counting are the ones that are lighter than the Hubble parameter  $H$ , which plays a role similar to the black hole temperature in de-Sitter space. In gauged M-flation model, we find the modified UV cutoff  $\Lambda$  to be few percent of  $M_{\text{pl}}$  due to hierarchical nature of isocurvature modes mass spectra. Although we focused our analysis on the gauged M-flation case, from the results of [1] and our discussions of section 4, it is obvious that a similar conclusion could be drawn for the ungauged case too.

The situation is, however, different in chaotic assisted models [6] or N-flation [7] in which all the isocurvature modes have masses smaller than the Hubble parameter. In this regard M-flation is more successful than these models, as it is UV safe. The problem of excursion beyond the cutoff remains an open question in those models. Of course, some assisted scenarios like multiple M5-brane inflation [26, 27] may survive, as in those models the physical field excursions are scaled by  $N^{-3/2}$ . Even though the background classical energy density need not be smaller than  $\Lambda^4$ , the gauged M-flation energy density happens to respect this bound. As discussed the effective mass parameter in the SU(2) sector (2.14) and the original M-flation action (2.1) are the same and there is no  $N$  scaling for the mass parameter. To have a successful inflation  $m \sim 10^{-6}M_{\text{pl}}$  and hence  $m/\Lambda \sim 10^{-4}$ . That is, the hierarchy of the mass parameter is reduced (improved) by two orders of magnitude compared to simple  $m^2\phi^2$  case.

We would also like to comment on the stability of the classical inflationary trajectory in gauged M-flation with the symmetry breaking potential, with respect to the quantum production of  $\Psi_i$  modes. A similar analysis for the case of ungauged  $\lambda\Phi^4$  M-flation has been carried out in [1], showing that backreaction of the large number of spectator fields during slow-roll inflation will always remain small. Following the line of arguments in section 6 of [1], one can show that a similar result holds for the symmetry breaking case, in both gauged and ungauged cases. In this respect, the situation in the gauged case is better, as there are no “zero modes” and the zero modes are replaced with more massive states in the vector field fluctuations (the zero modes are replaced by the “longitudinal” modes of massive vector fields). Being more massive, it is harder to excite these modes and hence their backreaction is also reduced. The species cutoff considerations actually help with the above stability argument, since the modes which are really excited are even less, modes with  $j > j_{\text{max}}$  (*cf.* (4.4)) will not enter into the backreaction analysis.

A large number of “light species” of isocurvature modes, besides the reduction of the effective gravitational cutoff discussed above, may give rise to the problem of “dominance of quantum fluctuations” of the isocurvature modes, effectively pushing the theory to the eternal inflation phase, rather than slow-roll inflation governed by the effective inflaton field. As was pointed out in [28], in N-flation if the number of the fields goes beyond  $N_c = M_{\text{pl}}^2/\bar{m}^2$ , the slow-roll phase of inflation disappears.  $\bar{m}^2$  is the r.m.s. of the masses of the fields that play role during inflation. As a result ordinary N-flation models suffer from this problem. In M-flation scenario, however, the hierarchical nature of masses in the directions orthogonal to the SU(2) sector saves the theory from this potential problem too. To see this let us note that eternal phase of inflation starts when the total classical displacement of the inflaton(s),  $\delta\phi_{\text{CL}}$ , becomes equal to its (their) quantum fluctuations,  $\delta\phi_{\text{QM}}$ , [29]. The total amount of

quantum fluctuations in M-flation is

$$\delta\phi_{\text{QM}} = \sum_{i=1} \frac{H}{2\pi}, \quad (5.1)$$

where  $i$  runs over all the modes that are lighter than the Hubble parameter,  $N_s$  for M-flation. On the other hand, the total amount of classical displacements is given by

$$\delta\phi_{\text{CL}} = \left| \frac{V'_0(\phi)}{3H^2} \right| = M_{\text{pl}}^2 \left| \frac{V'_0(\phi)}{V_0(\phi)} \right| \quad (5.2)$$

where  $V_0(\phi)$  is the collective potential which results from all the fields, eq.(2.14). For different regions of our M-flation scenario, one can show that  $\delta\phi_{\text{QM}}$  is subdominant to the  $\delta\phi_{\text{CL}}$ . For example, for  $\phi > \mu$ ,  $\delta\phi_{\text{QM}} \simeq 0.003M_{\text{pl}}$  and  $\delta\phi_{\text{CL}} \simeq 0.16M_{\text{pl}}$  around  $\phi_{\text{ini}} \simeq 43.5M_{\text{pl}}$ . Thus the problem of dominance of quantum fluctuations in N-flation will never occur in the case of M-flation, even if the number of D3-branes is increased.

It is also worthwhile to note that the gauged M-flation in the region  $\phi > \mu/2$  is a local attractor for any perturbation about the SU(2) sector trajectory. This could be seen from the fact that the mass squared of  $\alpha_j$ ,  $\beta_j$  and all the gauge modes, except for the  $j = 0$  gauge mode<sup>5</sup>, is positive around the  $\Psi_i = 0$  trajectory, if gauged M-flation happens in the region  $\mu/2 < \phi$ .<sup>6</sup> This guarantees that  $\Psi_i = 0$  is at least a local attractor and departures from the SU(2) sector will ultimately vanish. The rate of approaching the attractor is set by the ratio of  $\Psi_i$ -mass over the Hubble parameter and therefore deviations in the directions of “small group” of light modes will take  $H/m$  e-folds to reach the SU(2) attractor. However, the situation is still much better than the assisted N-flation in which “all” the orthogonal modes are lighter than the Hubble parameter. If the UV cutoff is at the Planck mass, the perturbations of the  $\Psi$ -modes that are heavier than the Hubble parameter will reach the SU(2) sector in much less than an e-fold.

Finally we comment that, although the background (2.3) is a solution to type IIB supergravity and one can consistently study dynamics of  $N$  D3-branes in this background, the directions transverse to branes are noncompact and hence the effective four dimensional Newton constant goes to zero. In order to have a precise string theory derivation of (2.1), we need to complete the background (2.3) by viewing it as a part of the geometry which has six compact directions of finite volume. This latter is conceptually similar to usual mobile

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<sup>5</sup> $j = 0$  gauge mode is massless and in the spectrum it corresponds to the U(1) sector of the gauge fields. Being in the U(1) sector, this massless mode does not couple to the scalar fields  $\Phi_i$ .

<sup>6</sup>For M-flation in the region  $\mu/2 < \phi < \mu$ , the lightest spectator is  $\alpha_0$ .  $M_{\alpha_0}^2$  is negative during about the first 34 e-folds of inflation, while all the other spectators have positive mass squared for the whole inflationary period. On the other hand, we note that the  $\alpha_0$  mode is nothing but the fluctuation of the effective inflaton  $\phi$  itself, and the negative mass squared for some period during initial phase of inflation is a generic feature of any hilltop inflation. Existence of this modes is hence not inducing any instability in the reduction to the SU(2) sector. On the other hand, M-flation in the region  $0 < \phi < \mu/2$ , besides the  $M_{\alpha_0}^2$ , the first  $\sim 93$   $\alpha$ -modes become tachyonic, at least for a while, during the 60 e-folds of inflation. Thus, in this M-flationary region, one expects perturbations around the SU(2) sector in these directions to grow, causing instability in the SU(2) sector inflationary trajectory.

brane inflationary models where the brane is moving in an AdS throat while the geometry away from the throat is  $R^4 \times CY3$ . Moreover, given this “completed” geometry one should make sure that the backreaction from the stack of  $N$  D3-branes would remain negligible. We will postpone the study of these issues and a precise string theory realization of M-flation to a future publication.

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## A Symmetry-breaking inflation

In this case the potential takes the form of a symmetry-breaking potential which has two global minima at  $\phi = 0$  and  $\phi = \mu$

$$V_0 = \frac{\lambda_{eff}}{4} \phi^2 (\phi - \mu)^2, \quad (A.1)$$

where  $\mu \equiv \sqrt{2}m/\sqrt{\lambda_{eff}}$ . The minimum at  $\phi = \mu$  corresponds to supersymmetric vacuum when  $N$  D3-branes blow up into a giant D5-brane in the presence of background RR field. The minimum at  $\phi = 0$ , on the other hand, corresponds to the trivial solution when matrices become commutative. If we allow the field  $\phi$  to take negative values, then the potential is symmetric under  $\phi \rightarrow -\phi + \mu$ .

Depending on the initial value of the inflaton field,  $\phi_i$ , the inflationary can take place in three different regions:

(a)  $\phi_i > \mu$

Suppose inflation starts when  $\phi_i > \mu$ . With  $N_e = 60$ ,  $\delta_H \simeq 2.41 \times 10^{-5}$  and  $n_s = 0.96$ , one obtains

$$\phi_i \simeq 43.57 M_{\text{pl}}, \quad \phi_f \simeq 27.07 M_{\text{pl}}, \quad \mu \simeq 26 M_P. \quad (A.2)$$

and

$$\lambda_{eff} \simeq 4.91 \times 10^{-14}, \quad m \simeq 4.07 \times 10^{-6} M_{\text{pl}}, \quad \kappa_{eff} \simeq 9.57 \times 10^{-13} M_{\text{pl}}. \quad (A.3)$$

Assuming that  $\lambda = 1$ , in order to obtain the desire value for  $\lambda_{eff}$ , one needs  $N = 54618$  D3-branes. The amount of excursion in the physical field space,  $\Delta\hat{\phi}$ , is

$$\frac{2(\phi_f - \phi_i)}{\sqrt{N(N^2 - 1)}} = 2.58 \times 10^{-6} M_{\text{pl}}. \quad (A.4)$$

The Hubble parameter in the beginning of inflation,  $H_i \equiv H(\phi_i) = 4.89 \times 10^{-5} M_{\text{pl}}$ . Maximum  $j$  for which  $\alpha$ -modes,  $\beta$ -modes and gauge modes are smaller than  $H_i^2$ , respectively, are

$$j_{max}^\alpha = 5, \quad j_{max}^\beta = 7, \quad j_{max}^g = 9. \quad (A.5)$$

The total number of species that would contribute to the cutoff is then  $N_s = 398$ . The UV cutoff,  $\lambda$ , is found to be

$$\Lambda = 5 \times 10^{-2} M_{\text{pl}}, \quad (\text{A.6})$$

which is much larger than the field displacement. The ratio  $\rho/\Lambda^4$  is  $1.14 \times 10^{-3}$ .

**(b)**  $\mu/2 < \phi_i < \mu$

To fit the observational constraints one obtains

$$\phi_i \simeq 23.5 M_{\text{pl}}, \quad \phi_f \simeq 35.03 M_{\text{pl}}, \quad \mu \simeq 36 M_P. \quad (\text{A.7})$$

and

$$\lambda_{eff} \simeq 7.18 \times 10^{-14}, \quad m \simeq 6.82 \times 10^{-6} M_{\text{pl}}, \quad \kappa_{eff} \simeq 1.94 \times 10^{-12} M_{\text{pl}}. \quad (\text{A.8})$$

Repeating the same analysis for this branch, one realizes that  $N = 48103$  D3-branes are needed. The amount of physical excursions of the field is  $2.185 \times 10^{-6} M_{\text{pl}}$ . Number of species that contribute to the cutoff is  $N_s = 195$  and the corresponding UV cutoff is  $\Lambda = 7.16 \times 10^{-2} M_{\text{pl}}$ . The ratio of energy density of the Universe during inflation to  $\Lambda^4$  is  $5.89 \times 10^{-4}$ .

**(c)**  $0 < \phi_i < \mu/2$

Due to symmetry  $\phi \rightarrow -\phi + \mu$  this inflationary region has the same properties as  $\mu/2 < \phi_i < \mu$  above and therefore the couplings have the same values as region **(b)**. However the mass expressions for the isocurvature modes do not enjoy the symmetry and therefore the numerics are a little bit different from case **(b)**. The total number of species contributing to the cutoff in this case is  $N_s = 794$  and the corresponding value for the UV cutoff is  $3.55 \times 10^{-2} M_{\text{pl}}$ . The ratio of the energy density of the Universe to  $\Lambda^4$  is  $9.76 \times 10^{-4}$ .

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